EE 230
Lecture 40

Data Converters

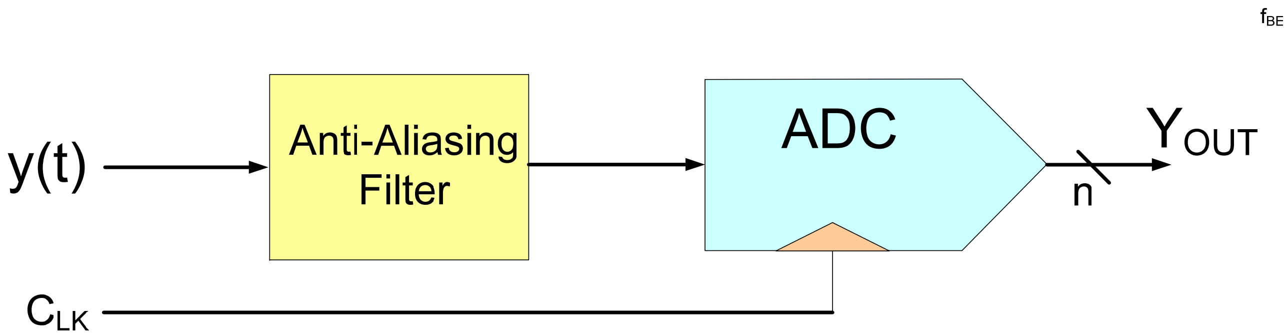
Amplitude Quantization

Quantization Noise

Review from Last Time:

Time Quantization

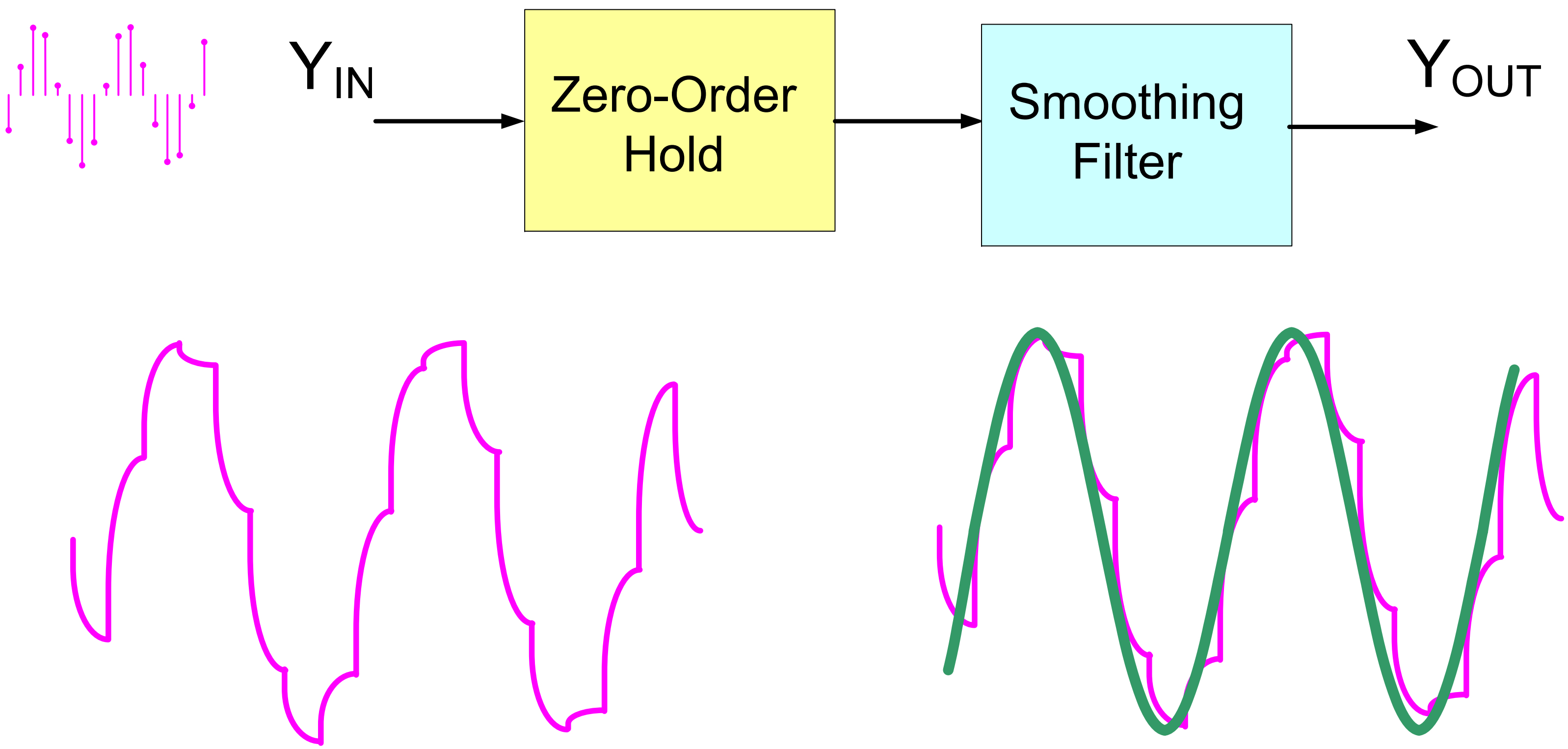
Typical ADC Environment



Review from Last Time:

Time Quantization

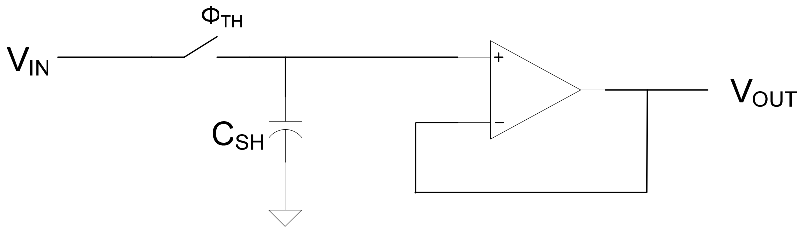
Analog Signal Reconstruction

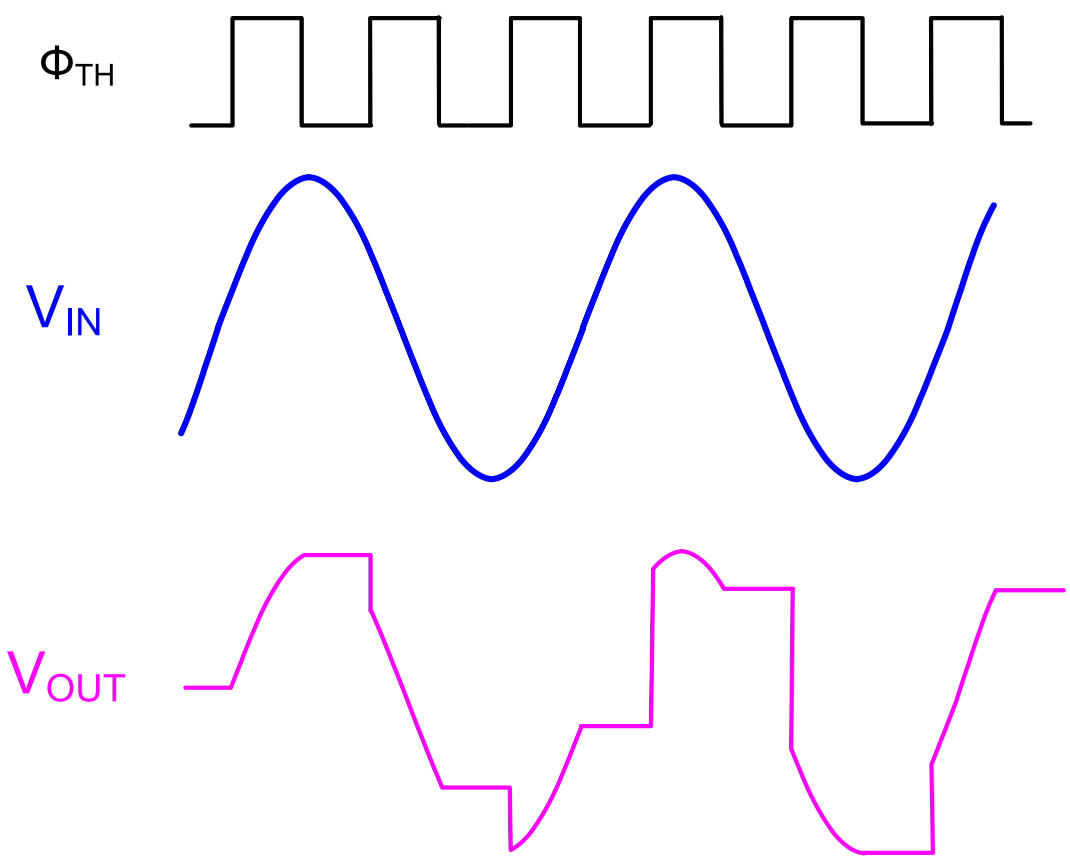


Smoothing filter removes some of the discontinuities in the output of the zero-order hold

Review from Last Time:

Track and Hold

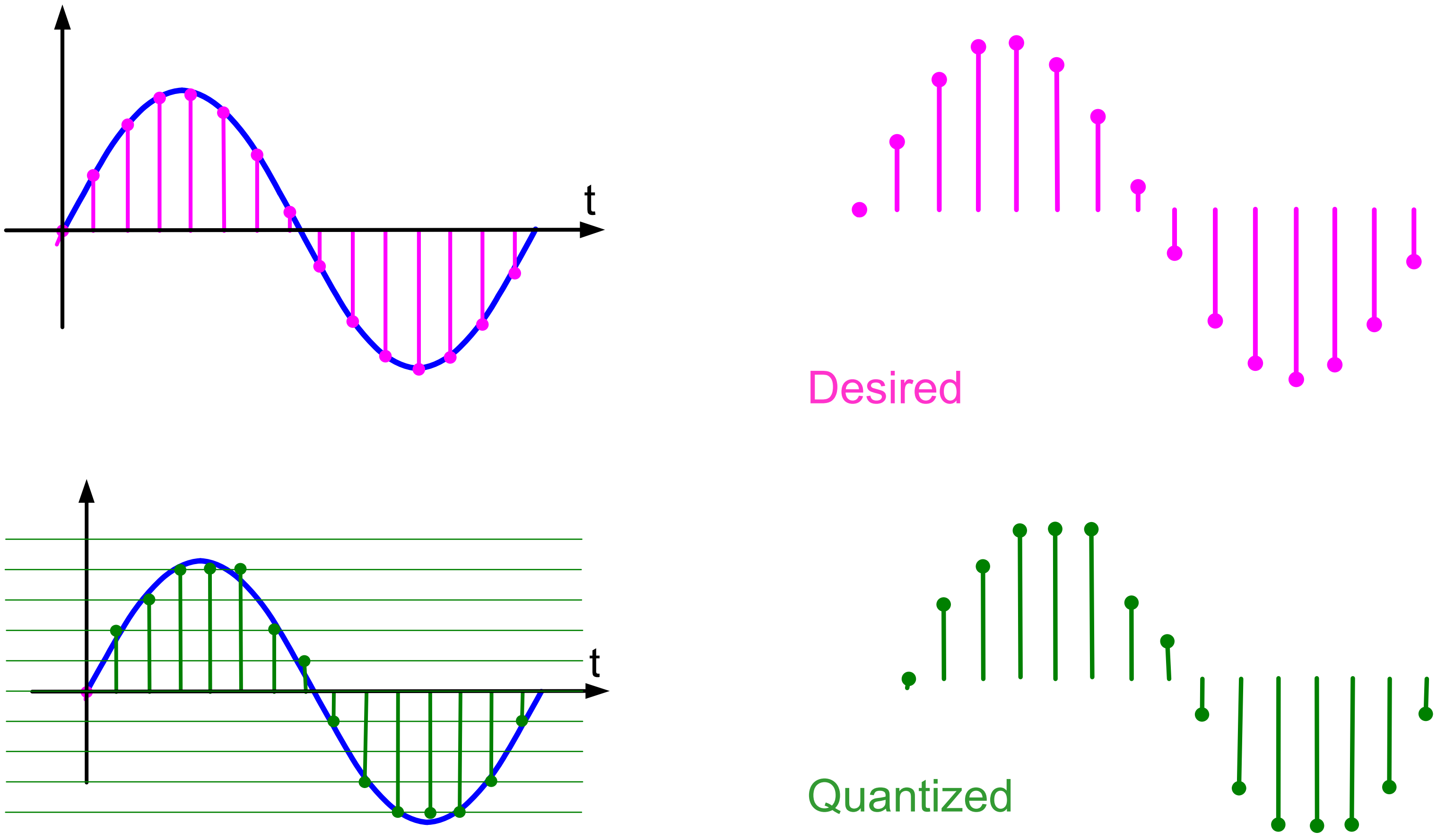




Review from Last Time:

Amplitude Quantization

Analog Signals at output of DAC are quantized
Digital Signals at output of ADC are quantized



Review from Last Time:

Amplitude Quantization

Unwanted signals in the output of a system are called noise.

Distortion

Smooth nonlinearities

Frequency attenuation

Large Abrupt Nonlinearities

Signals coming from other sources

Movement of carriers in devices

Interference from electrical coupling

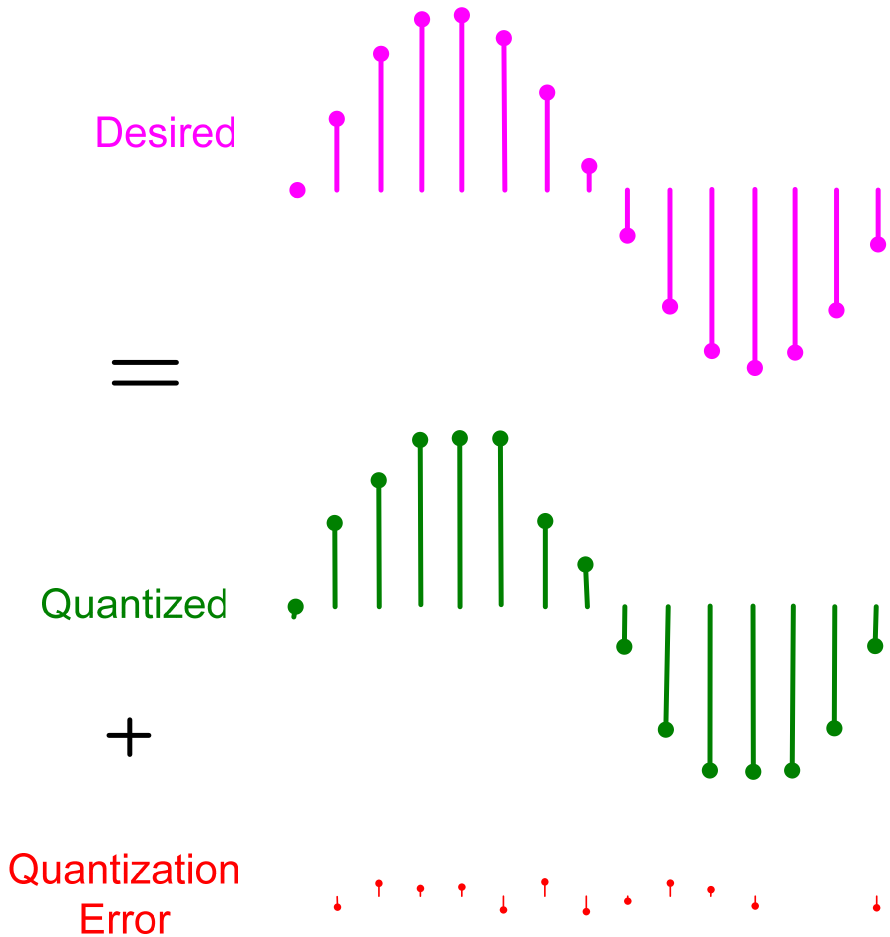
Interference from radiating sources

Undesired outputs inherent in the data conversion process itself

Review from Last Time:

Amplitude Quantization

How big is the quantization “noise” characterized?



Amplitude Quantization

Characterization of Quantization Noise

Quantization noise is usually specified in terms of the rms value of the quantization error expressed relative to the LSB

For convenience, consider the quantization noise for the following two waveforms that are as large as possible without without exceeding the input or output range of the data converter

a) triangle or saw tooth

b) sinusoidal

Amplitude Quantization

Characterization of Quantization Noise

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For convenience, consider the quantization noise for the following two waveforms that are as large as possible without without exceeding the input or output range of the data converter

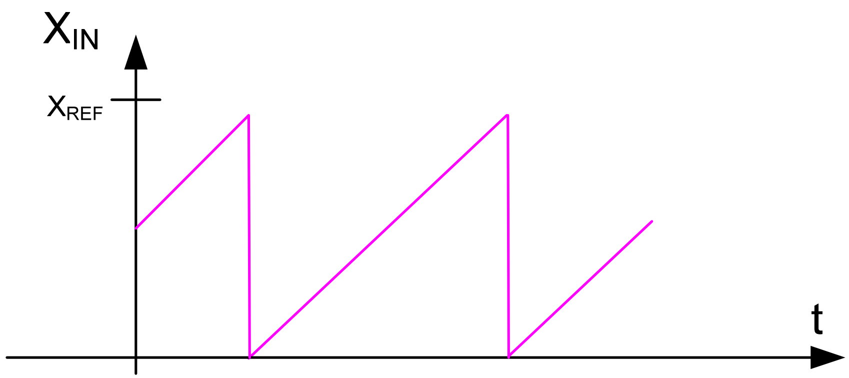
a) triangle or saw tooth

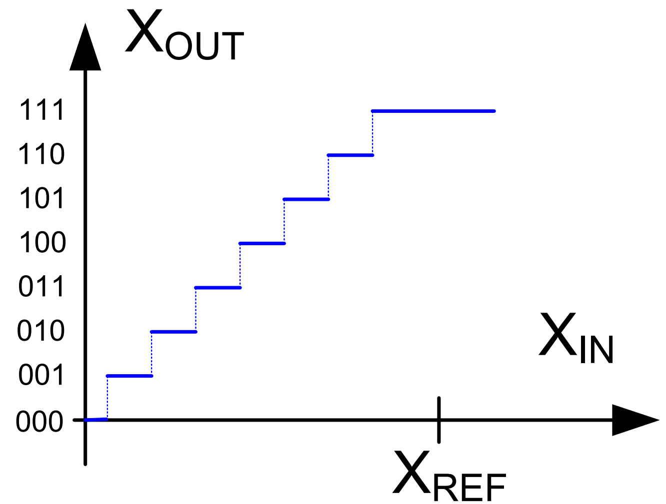
b) sinusoidal

Characterization of Quantization Noise

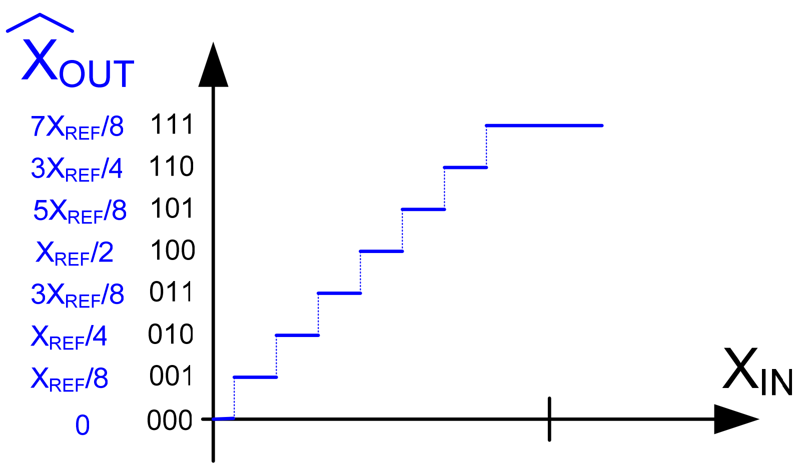
Saw tooth excitation

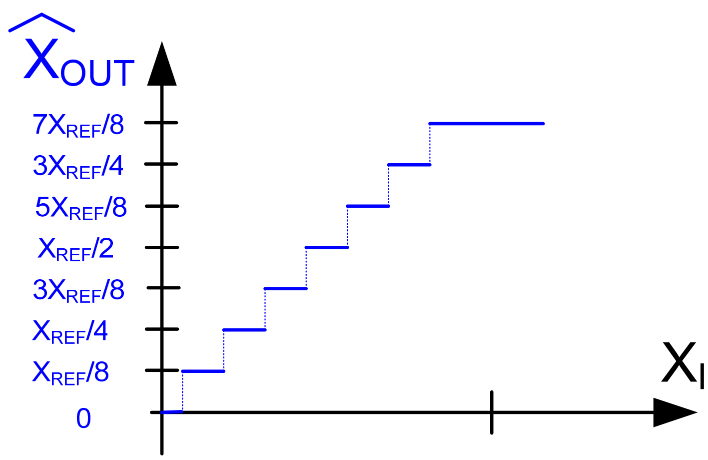
- Consider an ADC (flash)





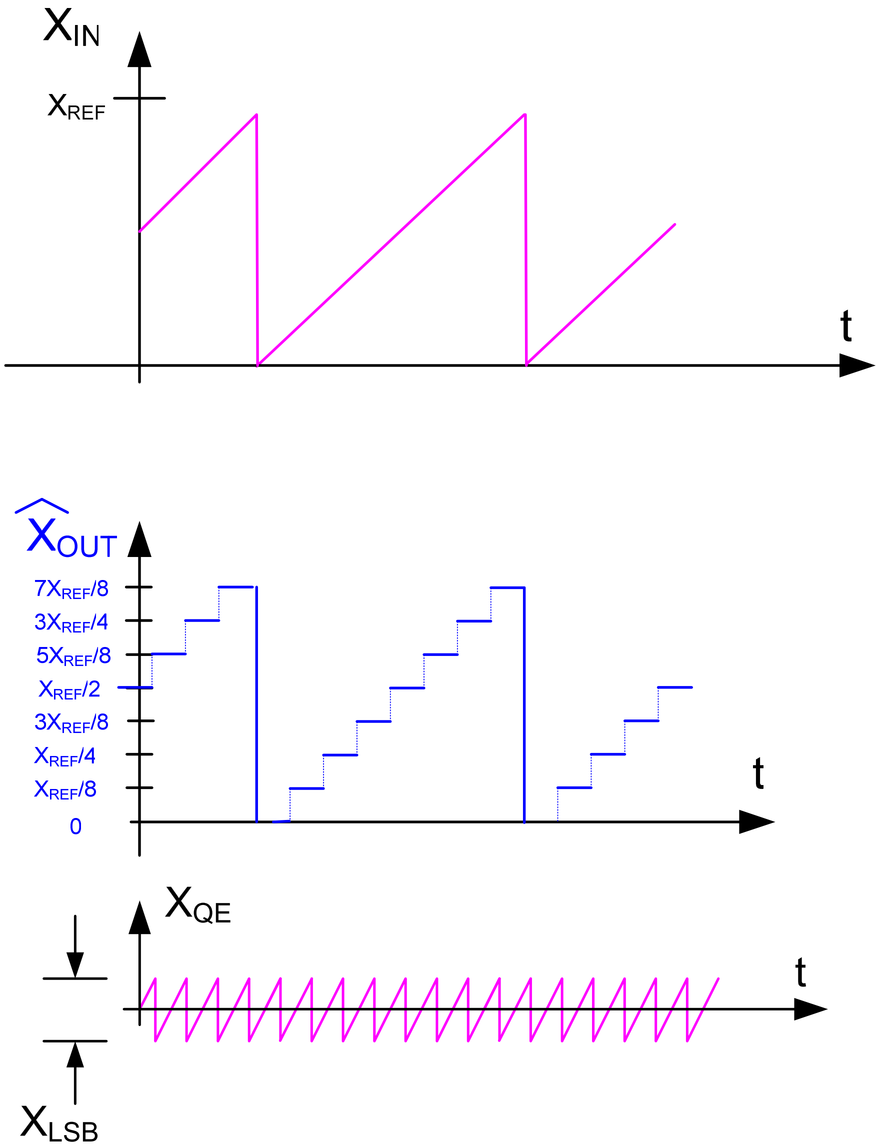
Assume first transition at $X_{REF}/(2^{(n+1)})$





Characterization of Quantization Noise

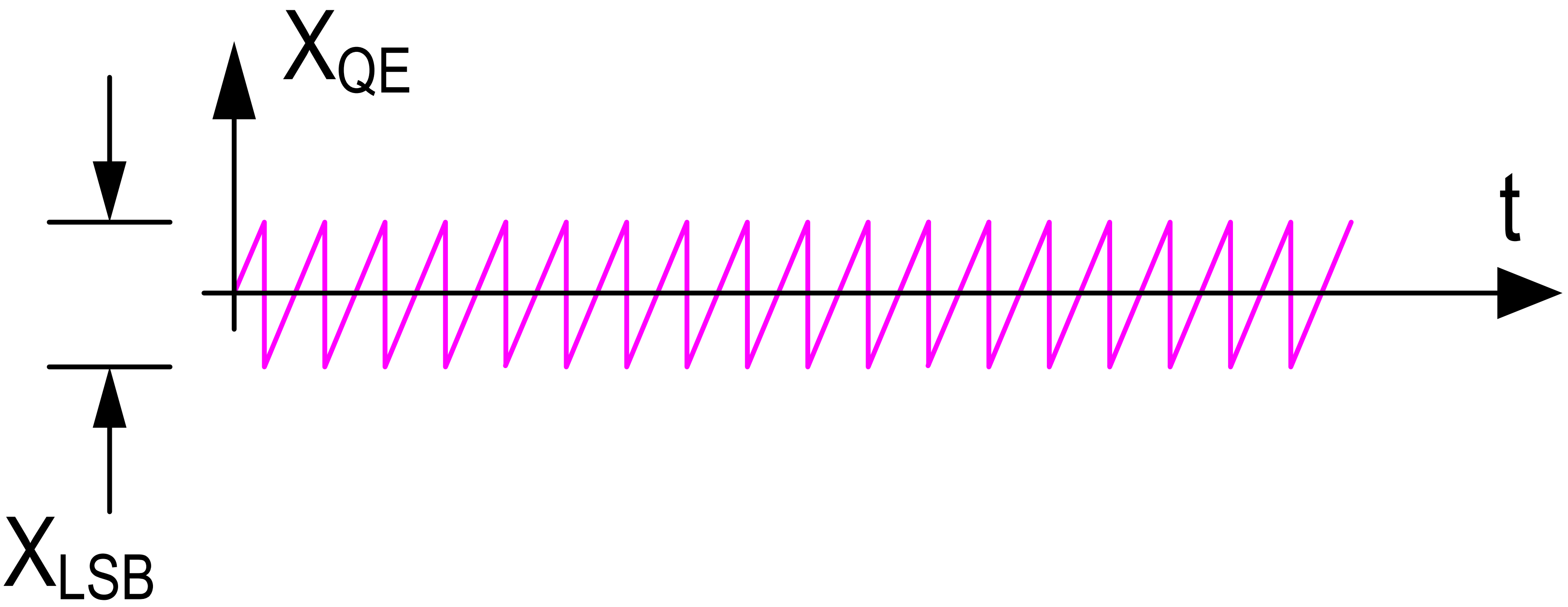
Saw tooth excitation



Characterization of Quantization Noise

Saw tooth excitation

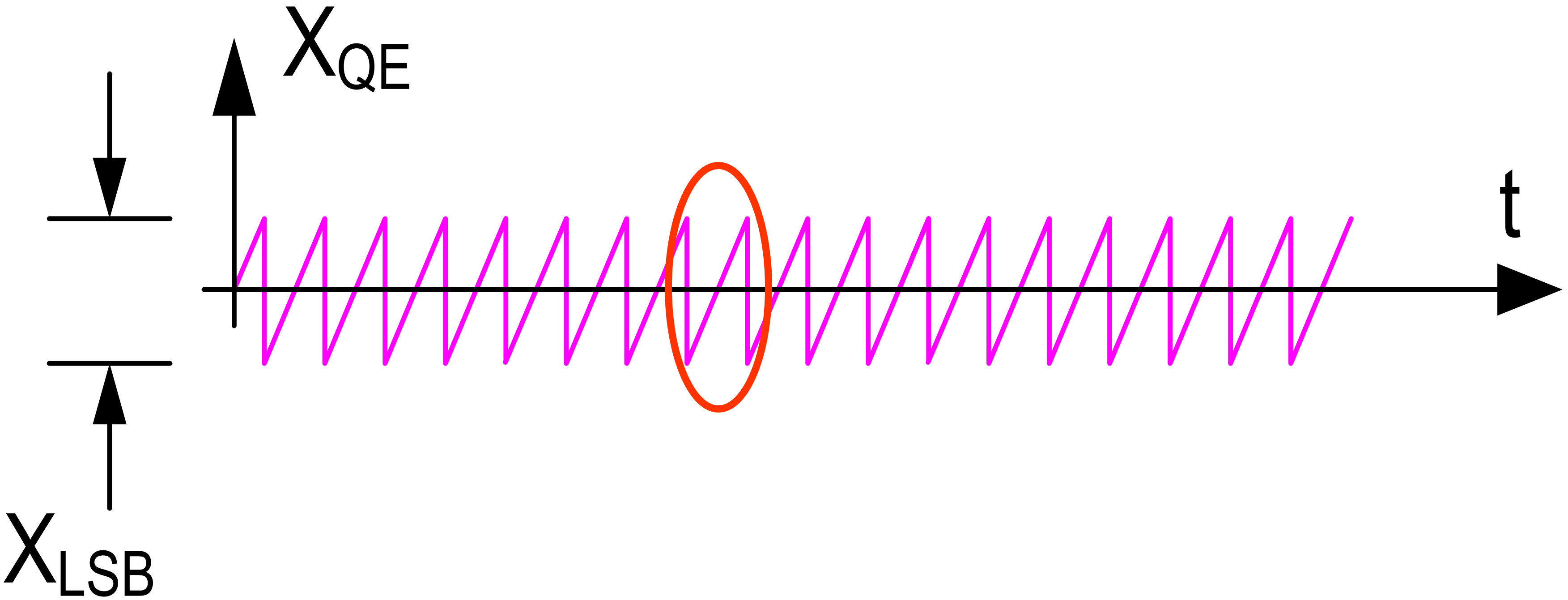
Error waveform:



Characterization of Quantization Noise

Saw tooth excitation

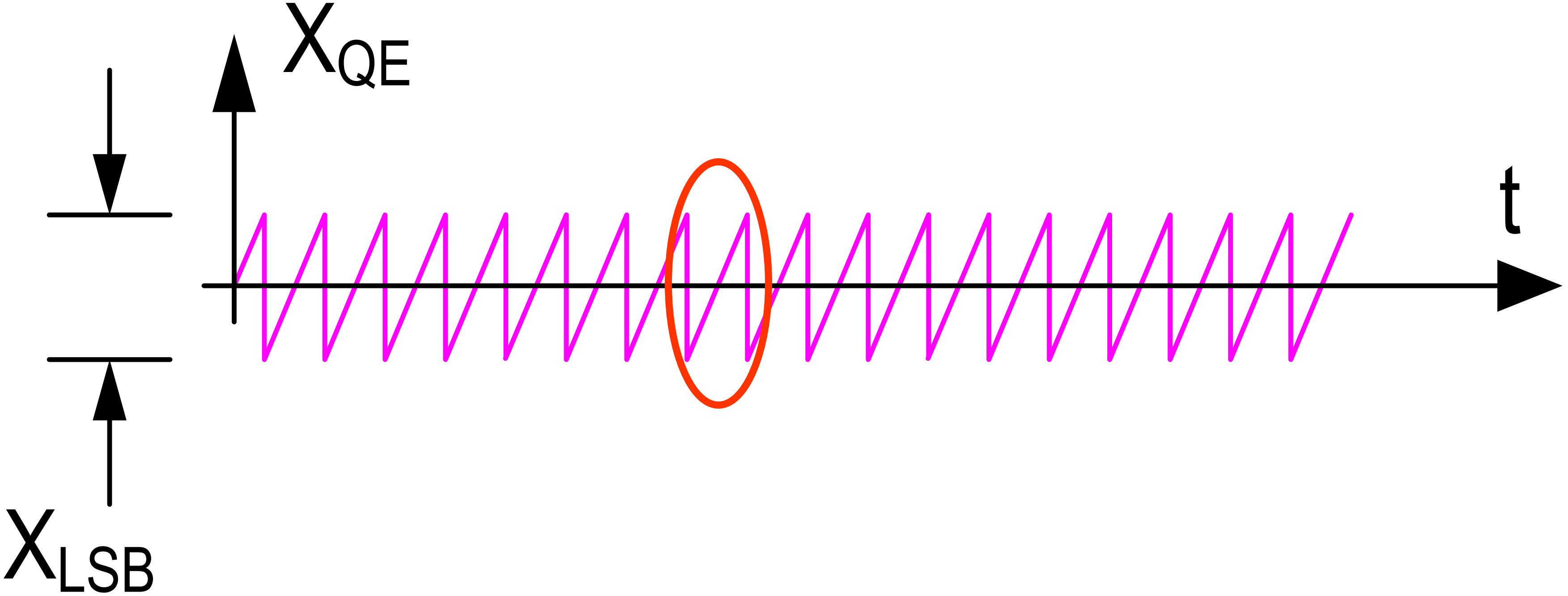
Error waveform:



Characterization of Quantization Noise

Saw tooth excitation

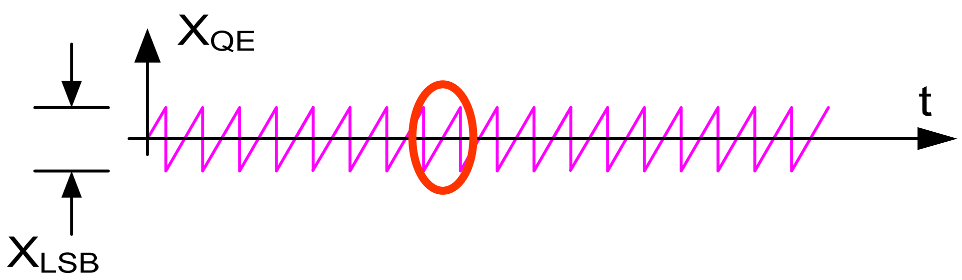
Error waveform:

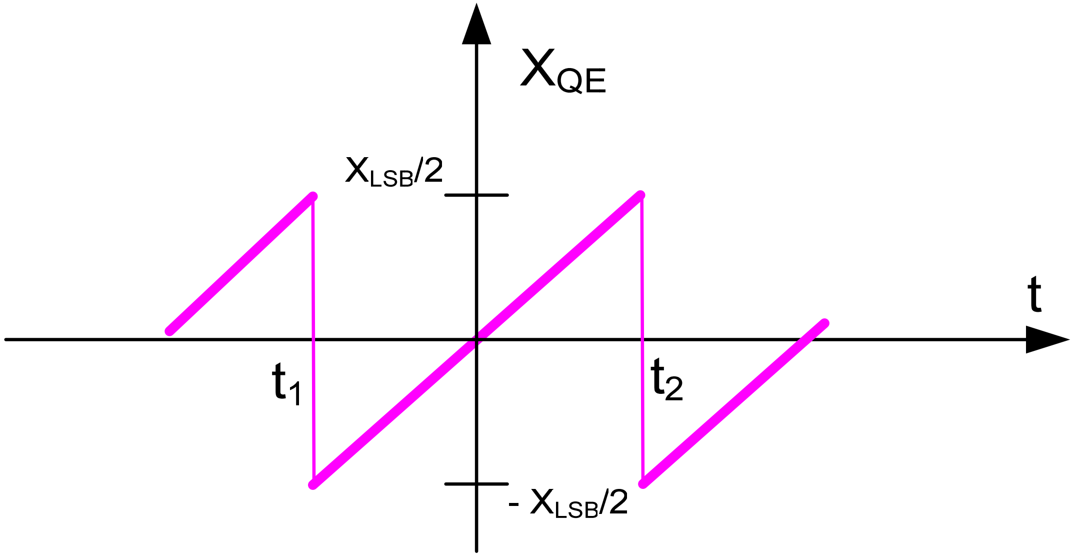


Characterization of Quantization Noise

Saw tooth excitation

Error waveform:





Since $x_{QE}(t)$ is periodic, the X_{QRMS} can be obtained by integrating $x_{QE}^2(t)$ over one period

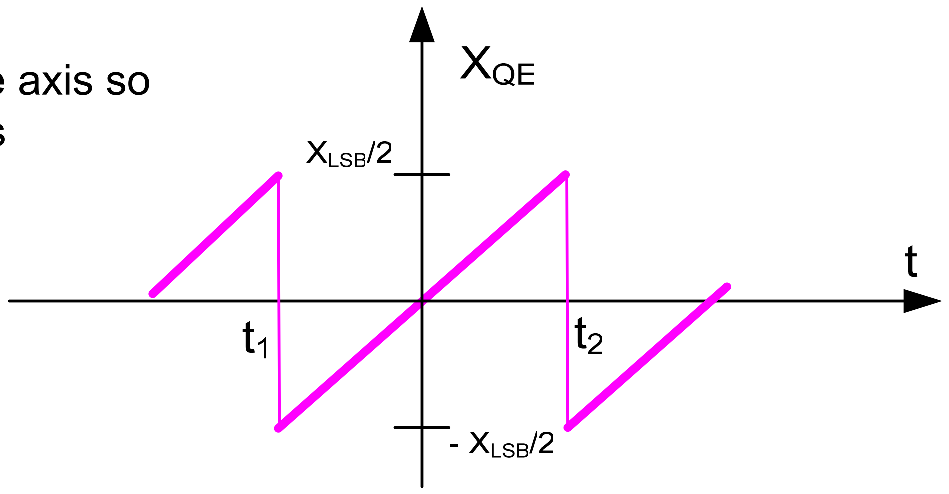
$$X_{QRMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_2} x^2_{QE}(t)\,dt}$$

Characterization of Quantization Noise

Saw tooth excitation

without loss of generality, can shift time axis so that X_{QE} is symmetric to the origin, thus

$$t_1 = -\frac{T}{2} \qquad t_2 = \frac{T}{2}$$



for $t_1 < t < t_2$, can express X_{QE} as

$$x_{QE}(t) = \frac{X_{LSB}}{T} t$$

thus

$$X_{QRMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_2} x^2_{QE}(t)\,dt} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2}\left[\frac{X_{LSB}}{T}\right]^2 t^2 dt}$$

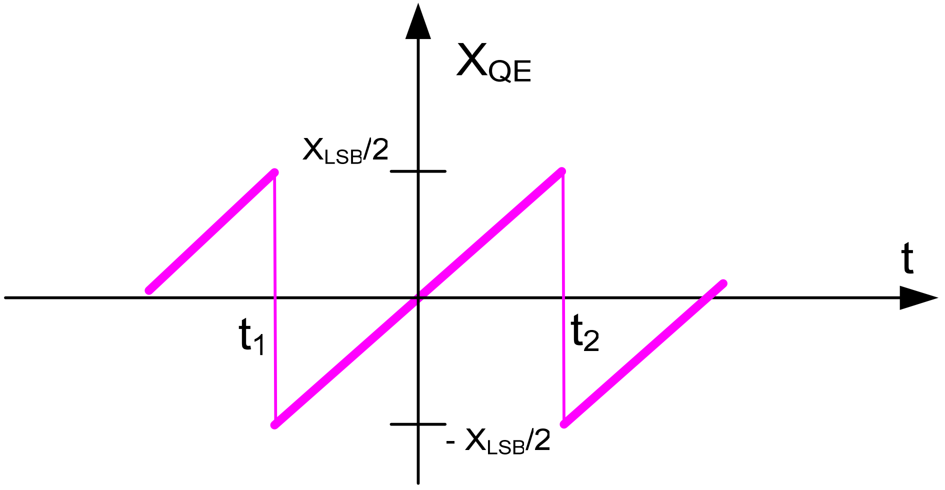
$$X_{QRMS} = \sqrt{\frac{X^2_{LSB}}{T^3}\int_{-T/2}^{T/2} t^2 dt} = \sqrt{\frac{X^2_{LSB}}{T^3}\left.\frac{t^3}{3}\right|_{-T/2}^{T/2}} = \frac{X_{lSB}}{\sqrt{12}}$$

Characterization of Quantization Noise

Saw tooth excitation

$$X_{QRMS} = \frac{X_{lSB}}{\sqrt{12}}$$

$$X_{QRMS} \simeq \frac{1}{3} X_{lSB}$$



Is this quantization noise signal small or large?

Whether this is viewed as being large or small depends upon how the noise relates to the signal !

Signal to Noise Ratio

$$SNR = \frac{Signal}{Noise}$$

Signal and Noise are generally expressed in terms of their RMS current or voltage values

$$SNR = \frac{X_{SIG\text{-}RMS}}{X_{NOISE\text{-}RMS}}$$

Or, sometimes in terms of their "Power" values – assumed driving resistive loads

$$SNR_P = \frac{P_{SIG\text{-}RMS}}{P_{NOISE\text{-}RMS}}$$

Thus

$$SNR_P = \frac{X^2_{SIG\text{-}RMS}}{X^2_{NOISE\text{-}RMS}} = SNR^2$$

Signal to Noise Ratio

Signal and Noise often expressed in dB

$$SNR_{dB} = 20\log_{10}\left(\frac{X_{SIG\text{-}RMS}}{X_{NOISE\text{-}RMS}}\right)$$

$$SNR_{P\text{-}dB} = 10\log_{10}\left(\frac{P_{SIG}}{P_{NOISE}}\right)$$

But

$$SNR_{P\text{-}dB} = 10\log_{10}\left(\frac{X^2_{SIG\text{-}RMS}}{X^2_{NOISE\text{-}RMS}}\right) = 20\log_{10}\left(\frac{X_{SIG\text{-}RMS}}{X_{NOISE\text{-}RMS}}\right) = SNR_{dB}$$

$$SNR_{P\text{-}dB} = SNR_{dB}$$

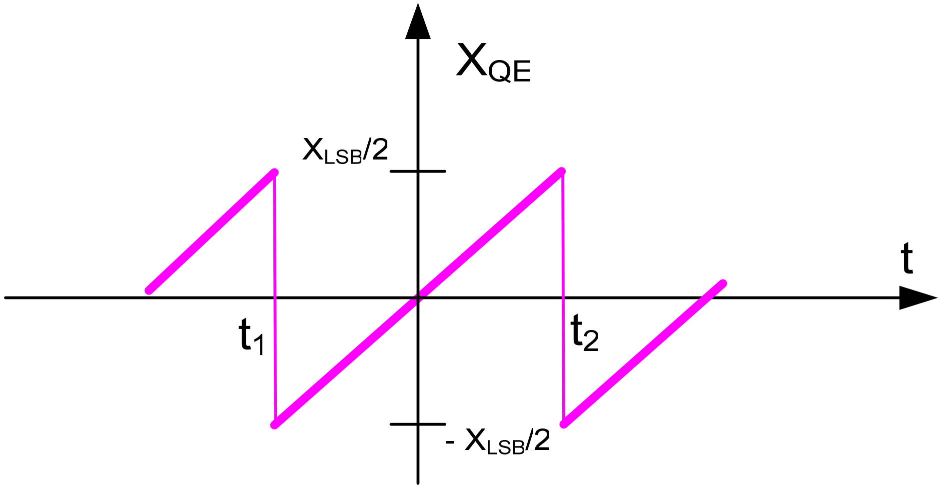
Often subscripts are dropped and will not cause a problem if in dB

$$SNR = SNR_{P\text{-}dB} = SNR_{dB}$$

Characterization of Quantization Noise

Saw tooth excitation

$$X_{QRMS} = \frac{X_{lSB}}{\sqrt{12}}$$



What is the SNR of a data converter ?

$$SNR = \frac{X_{SIG\text{-}RMS}}{X_{NOISE\text{-}RMS}}$$

What is $X_{SIG\text{-}RMS}$?

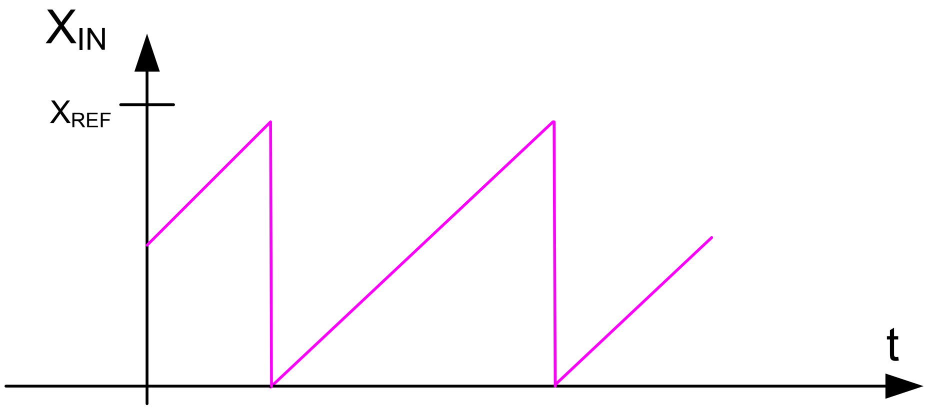
Characterization of Quantization Noise

Saw tooth excitation

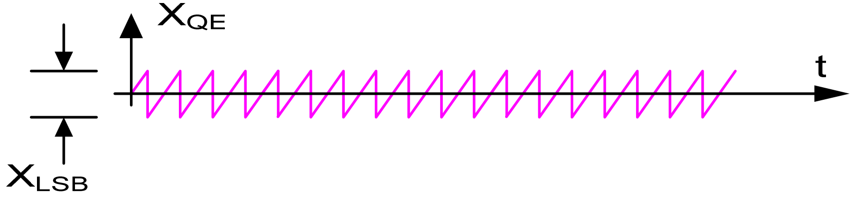
$$X_{QRMS} = \frac{X_{lSB}}{\sqrt{12}}$$

$$SNR = \frac{X_{SIG\text{-}RMS}}{X_{NOISE\text{-}RMS}}$$

What is $X_{SIG\text{-}RMS}$?



$$X_{SIG-RMS} = \frac{X_{REF}}{\sqrt{12}}$$



Characterization of Quantization Noise

Saw tooth excitation

$$X_{QRMS} = \frac{X_{lSB}}{\sqrt{12}} \qquad\qquad X_{SIG-RMS} = \frac{X_{REF}}{\sqrt{12}}$$

$$\text{SNR}=\frac{X_{REF}/\sqrt{12}}{X_{LSB}/\sqrt{12}} = \frac{X_{REF}}{X_{LSB}}$$

but $\quad X_{LSB} = \dfrac{X_{REF}}{2^n}$

Thus, the SNR for a data converter due to only the quantization noise is

$$\text{SNR} = \frac{1}{2^n} \qquad \text{or} \qquad \text{SNR}_{\text{dB}} = -\text{n}\log_{10}(2) = -6.02\text{n}$$

Characterization of Quantization Noise

Saw tooth excitation

SNR for a data converter due to only the quantization noise:

$$\text{SNR} = \frac{1}{2^n}$$

$$\text{SNR}_{\text{dB}} = -6.02 \bullet \text{n}$$

Amplitude Quantization

Characterization of Quantization Noise

Quantization noise is usually specified in terms of the rms value of the quantization error expressed relative to the LSB

For convenience, consider the quantization noise for the following two waveforms that are as large as possible without without exceeding the input or output range of the data converter

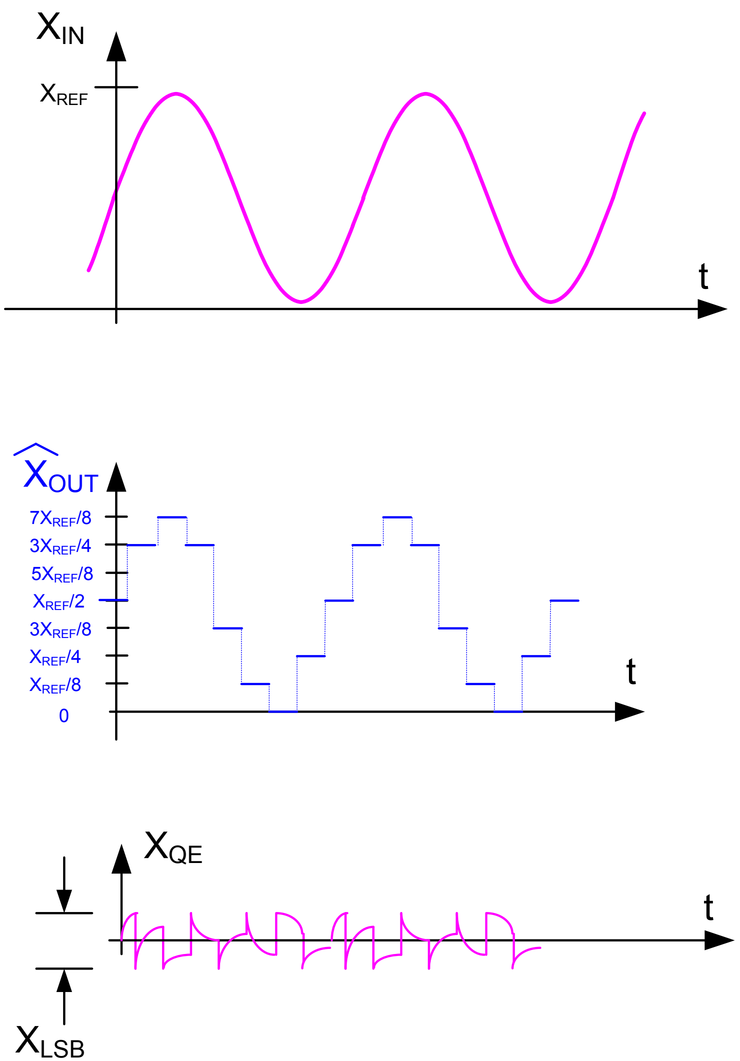
a) triangle or saw tooth

b) sinusoidal

Characterization of Quantization Noise

Sinusoidal excitation

- Consider an ADC



Quantization noise is difficult to analytically characterize

Still need RMS value of $X_{QE}(t)$

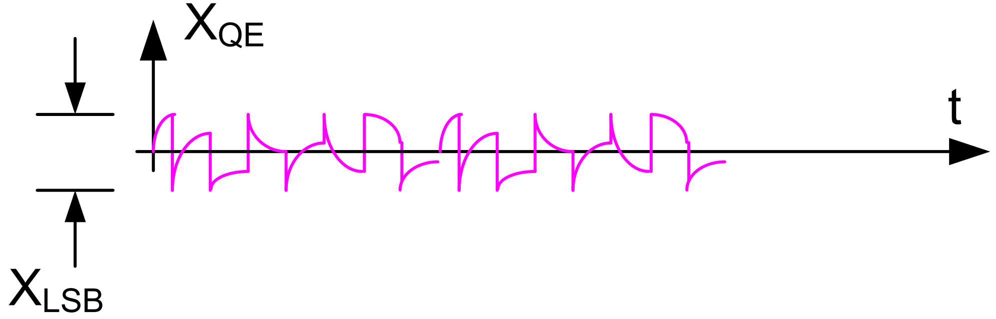
Will consider error in interpreted output

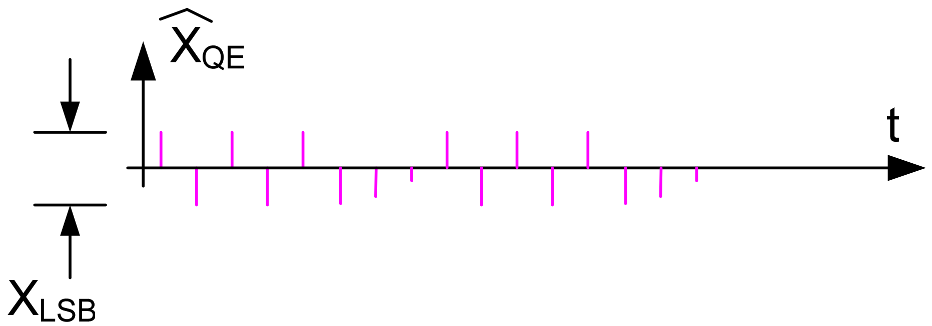
Characterization of Quantization Noise

Sinusoidal excitation

- Consider an ADC

Will consider error in interpreted output

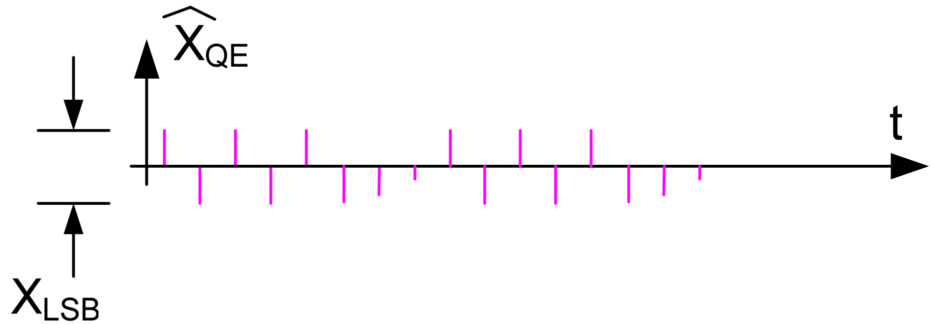




Characterization of Quantization Noise

Sinusoidal excitation

- Consider an ADC (clocked)



Theorem: If n(t) is a random process, then $V_{RMS} \cong \sqrt{\sigma^2 + \mu^2}$

provided that the RMS value is measured over a large interval where the parameters σ and μ are the standard deviation and the mean of <n(kT)>

This theorem can thus be represented as

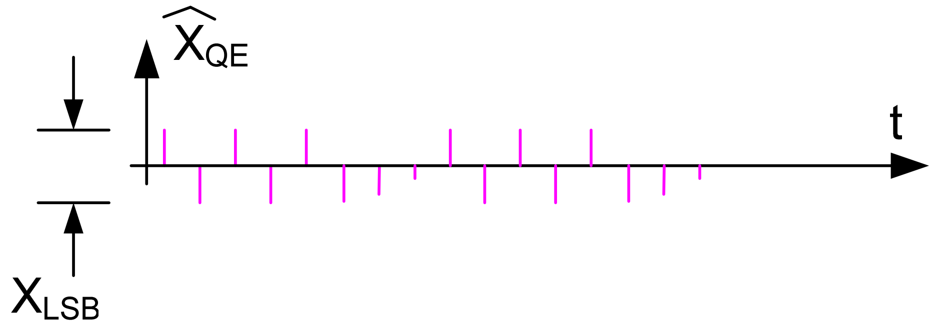
$$V_{RMS} \cong \sqrt{\frac{1}{T_L}\int_{t_1}^{t_1+T_L} n^2(t)\,dt} \cong \sqrt{\sigma^2 + \mu^2}$$

where T is the sampling interval and T_L is a large interval

Characterization of Quantization Noise

Sinusoidal excitation

- Consider an ADC



The quantization noise samples of the ADC output are approximately uniformly distributed between in the interval [-X_{LSB}/2 , X_{LSB}/2]

$$<n(kT)> \sim \mathrm{U}[-X_{LSB}/2\ ,\ X_{LSB}/2]$$

A random variable that is U[a,b] has distribution parameters μ and σ given by

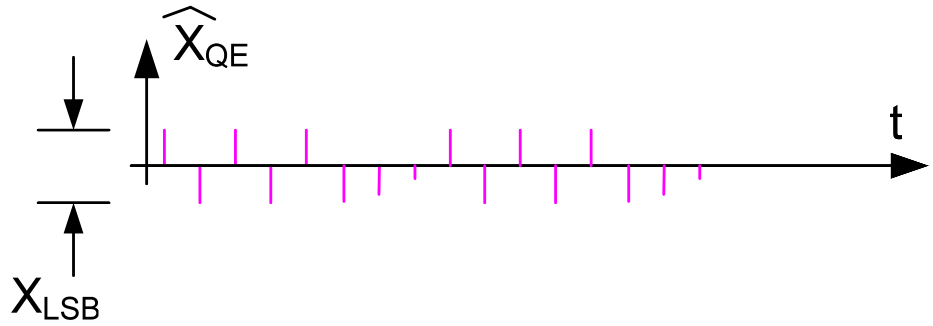
$$\mu = \frac{A + B}{2} \qquad \sigma = \frac{B - A}{\sqrt{12}}$$

thus, the random variable n(kT) has distribution parameters

$$\mu = 0 \qquad \sigma = \frac{X_{LSB}}{\sqrt{12}}$$

Characterization of Quantization Noise

Sinusoidal excitation



- Consider an ADC

n(kT) parameters

$\mu = 0 \qquad \sigma = \frac{X_{LSB}}{\sqrt{12}}$

It thus follows form the previous Theorem that

$$X_{Q\text{-}RMS} \cong \sqrt{\sigma^2 + \mu^2}$$

$$X_{Q\text{-}RMS} \cong \sqrt{\sigma^2} = \sigma$$

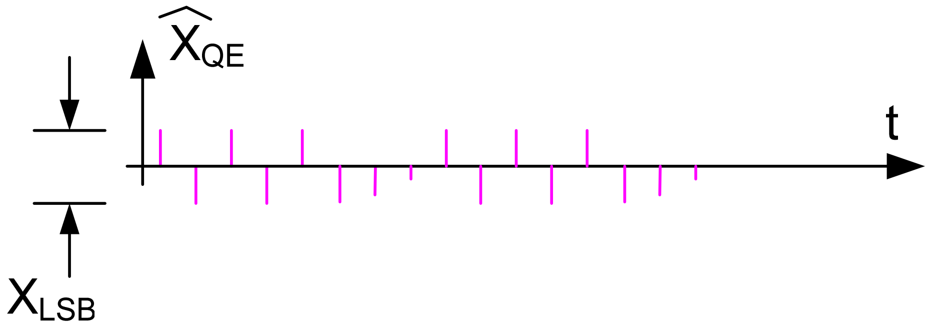
$$X_{Q\text{-}RMS} \cong \frac{X_{LSB}}{\sqrt{12}}$$

Note that this is the same quantization noise voltage as was obtained with the triangular excitation !

Characterization of Quantization Noise

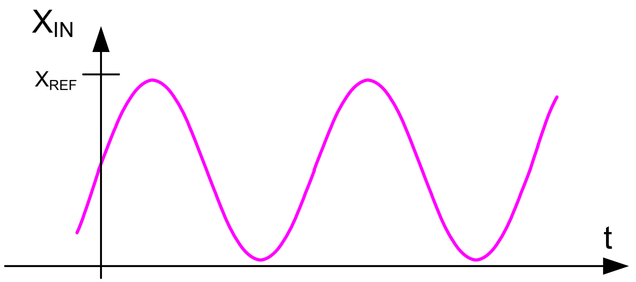
Sinusoidal excitation

- Consider an ADC



$$X_{Q\text{-}RMS} \cong \frac{X_{LSB}}{\sqrt{12}}$$

What is the SNR ?



$$X_{SIG}(t) = \frac{X_{REF}}{2}\sin(\omega t + \theta) + \frac{X_{REF}}{2}$$

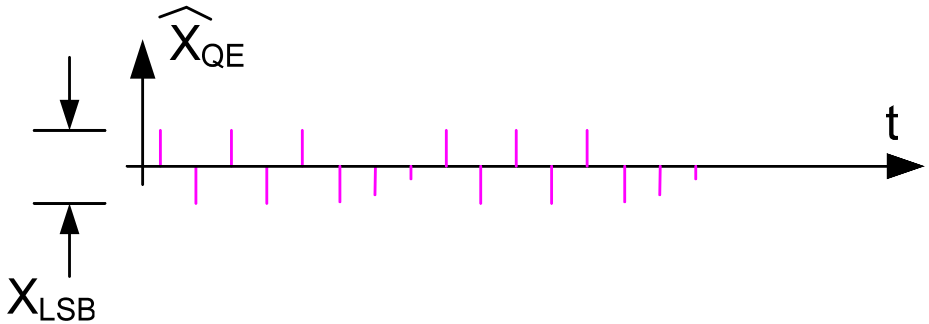
$$X_{SIG-RMS} = \frac{1}{\sqrt{2}}\left|\frac{X_{REF}}{2}\right| = \frac{X_{REF}}{2\sqrt{2}}$$

Observe that the RMS value for the sinusoidal signal differs from that of the triangular signal $X_{SIG-RMS} = \frac{X_{REF}}{\sqrt{12}} = \frac{X_{REF}}{2\sqrt{3}}$

Characterization of Quantization Noise

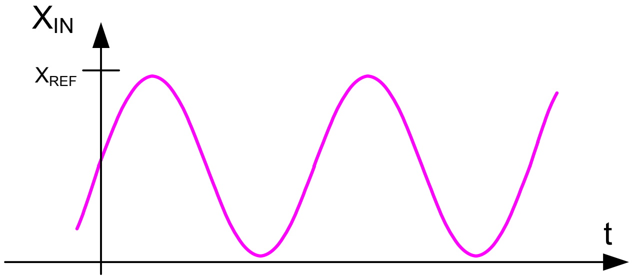
Sinusoidal excitation

- Consider an ADC



$$X_{Q\text{-}RMS} \cong \frac{X_{LSB}}{\sqrt{12}}$$

$$X_{SIG-RMS} = \frac{X_{REF}}{2\sqrt{2}}$$



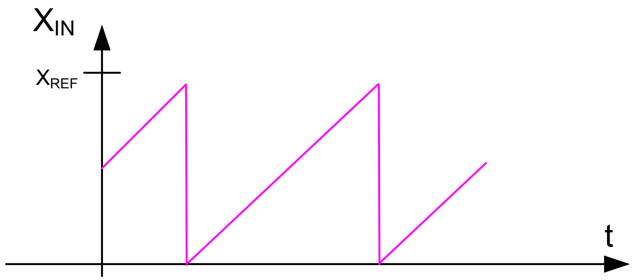
What is the SNR ?

$$\text{SNR}=\frac{X_{REF}\big/2\sqrt{2}}{X_{LSB}\big/\sqrt{12}} = \sqrt{\frac{3}{2}}\frac{X_{REF}}{X_{LSB}} = 1.225\frac{X_{REF}}{X_{LSB}} = 1.225\frac{X_{REF}}{X_{REF}/2^n} = 1.225 \bullet 2^n$$

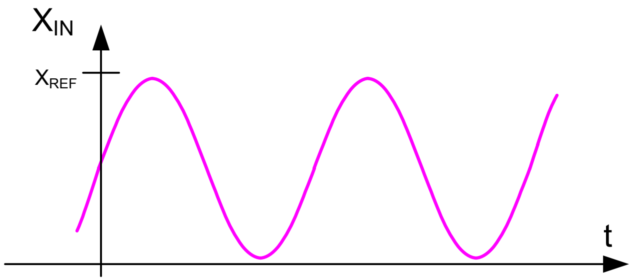
$$\text{SNR}_{dB}=20\log_{10}\left(1.225\frac{X_{REF}}{X_{LSB}}\right) = 20\log_{10}\left(1.225\frac{X_{REF}}{X_{REF}/2^n}\right) = 6.02n + 1.76$$

Characterization of Quantization Noise

Saw tooth excitation



Sinusoidal excitation



$$X_{Q\text{-}RMS} \cong \frac{X_{LSB}}{\sqrt{12}}$$

$$SNR = 2^n$$

$$SNR_{dB} = 6.02n$$

$$SNR = 1.225 \bullet 2^n$$

$$SNR_{dB} = 6.02n + 1.76$$

Although derived for an ADC, same expressions apply for DAC
SNR for saw tooth and for triangle excitations are the same
SNR for sinusoidal excitation larger than that for saw tooth by 1.76dB
SNR will decrease if input is not full-scale
Equivalent Number of Bits (ENOB) often given relative to quantization noise SNR_{dB}
Remember – quantization noise is inherent in an ideal data converter!

Recall: Unwanted signals in the output of a system are called <u>noise.</u>

Distortion

Smooth nonlinearities

Frequency attenuation

Large Abrupt Nonlinearities

Signals coming from other sources

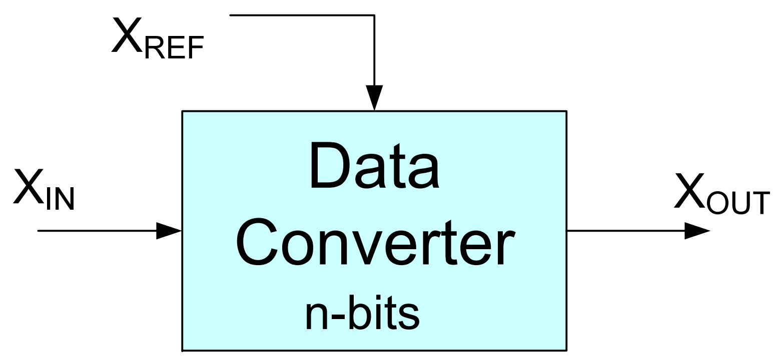
Interference from electrical coupling

Movement of carriers in devices

Interference from radiating sources

Undesired outputs inherent in the data conversion process itself

Equivalent Number of Bits (ENOB)



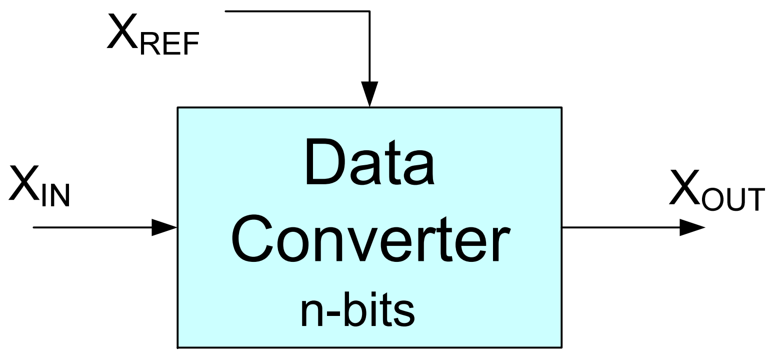
Often other sources of noise are present in a data converter and often the noise or other nonidealities in the data converter result in errors that are larger than 1/3 X_{LSB} and in many cases even much larger than X_{LSB}

ENOB often a figure of merit that is used to more effectively characterize the real resolution of a data converter <u>if a full-scale sinusoidal input were applied</u>

$SNR_{dB,ACT}$: Actual Signal to Noise Ratio

$$SNR_{dB,ACT} = 6.02n_{EFF} + 1.76$$

Equivalent Number of Bits (ENOB)



$$SNR_{dB,ACT} = 6.02 n_{EFF} + 1.76$$

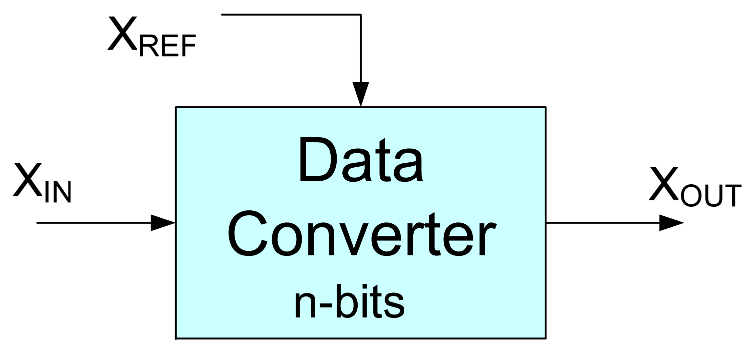
$$n_{EFF} = \frac{SNR_{dB,ACT} - 1.76}{6.02}$$

Often simply stated as

$$ENOB = \frac{SNR - 1.76}{6.02}$$

Observe: ENOB is not dependent upon n

Equivalent Number of Bits (ENOB)



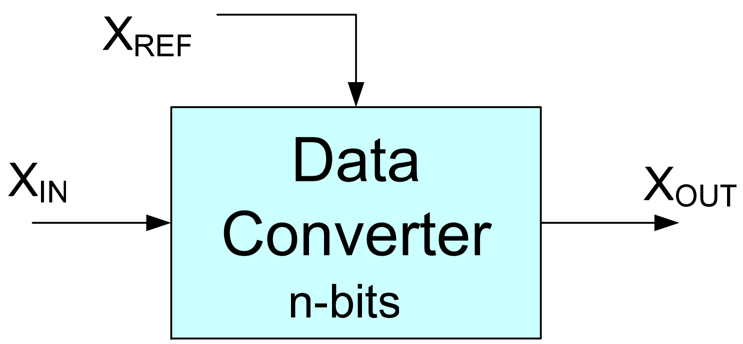
$$\mathrm{ENOB} = \frac{\mathrm{SNR} - 1.76}{6.02}$$

Often distortion is also included in the "noise" category and ENOB is expressed as

$$\mathrm{ENOB} = \frac{\mathrm{SNDR} - 1.76}{6.02}$$

where SNDR is the signal to (noise + distortion) ratio

Equivalent Number of Bits (ENOB)



$$\text{ENOB} = \frac{\text{SNR - 1.76}}{6.02} \qquad\qquad \text{ENOB} = \frac{\text{SNDR - 1.76}}{6.02}$$

These definitions of ENOB are based upon noise or noise and distortion

Some other definitions of ENOB are used as well – e.g. if one is only interested in distortion, an ENOB based upon distortion can be defined.

ENOB is useful for determining whether the number of bits really being specified is really useful

Example: The noise of a 12-bit ADC with V_{REF}=5V was measured to be 8.5mV.

a) What is the quantization noise of this 12-bit ADC (in V_{RMS})?
b) What is the ENOB?

Quantization noise:

$$X_{QRMS} \simeq \frac{1}{3} X_{lSB}$$

$$X_{QRMS} \simeq \frac{1}{3} \frac{V_{REF}}{2^{12}} = 0.41mV$$

Example: The SNR of a 12-bit ADC with V_{REF}=5V was measured to be 8.5mV.

a) What is the quantization noise of this 12-bit ADC (in V_{RMS})?
b) What is the ENOB?

Quantization noise: $$X_{QRMS} \simeq \frac{1}{3}\frac{V_{REF}}{2^{12}} = 0.41mV$$

ENOB: $$\text{ENOB} = \frac{\text{SNR - 1.76}}{6.02}$$

$$\text{SNR} = \frac{\frac{V_{REF}}{2\sqrt{2}}}{8.5\text{mV}} \qquad \text{SNR} = \frac{\frac{5\text{V}}{2\sqrt{2}}}{8.5\text{mV}} = \frac{1.77\text{V}}{8.5\text{mV}} = 208.2$$

$$\text{SNR}_{\text{dB}} = 20\log_{10}\left(SNR\right) = 20\log_{10}\left(208.2\right) = 46.4dB$$

$$\text{ENOB} = \frac{46.4\text{ - }1.76}{6.02} = 7.41$$

Note: In this application, an 8-bit ADC would give about the same SNR performance!

Engineering Issues for Using Data Converters

1. Inherent with Data Conversion Process

- Amplitude Quantization
- Time Quantization

(Present even with Ideal Data Converters)

2. Nonideal Components

- Uneven steps
- Offsets
- Gain errors
- Response Time
- Noise

(Present to some degree in all physical Data Converters)

How do these issues ultimately impact performance ?

End of Lecture 40